DEMONSTRATIONS OF SPATIAL MULTIPLEXING THAT ACCOMPANY:
Spatial multiplexing: Solving information bottlenecks in real neural systems and the origin of brain rhythms.

Nilsson, TH (2014) *International Journal of Adaptive, Resilient and Automatic Systems, 5*, (in press).

APPENDIX A: OPERATION OF A SIMPLE CONVERGENT-DIVERGENT NETWORK

To start, go to the author's University of Prince Edward Island web page: http://www.islandscholar.ca/fedora/repository/ir%3Anilsson . Open the file "Spatial Multiplexing". Download into a Corel QuattroPro spreadsheet the file *LinC-D2to1.qpw*.

On the CONVERGENT side of the spreadsheet in *Column* A starting at A4, the numbers 1 ,2, . . ., 22 label a linear array of inputs in *Column* C at C8, C10, ..., C50. Note the "1" in INPUT #6 at C20. *Column* E contains the convergent branching calculations. For example, click on E21 which has the value "2". This equals INPUT C18, plus twice the INPUT at C 20, plus twice the INPUT at C22, plus the INPUT at C24 (=C18+2*C20+2*C22+C24). This calculation represents the 2-and-1 branching shown in Figure 3.

To the right of the green pathway symbols, cells in *Column* I are set equal to the values in their corresponding cells in *Column* E. These are the only connection between the left and right sides of the spreadsheet.

On the DIVERGENT side of the spreadsheet, *Column* K contains the results of the divergent calculations based on the reverse 2-and-1 branch weights from cells in *Column* I. For example: K20 equals I17 plus twice I21. Calculations in *Column* L identify the local maximum value in *Column* I. Whichever *Column* M cell lies on the same row as the local maximum, divides, divides the value of the divergent result in *Column* K at that row by 5 to obtain an output value that equals the input.

That is all that is needed to make a linear convergent-divergent network that transmits a signal from any of *n* inputs along *n*/2 connections to *n* outputs while conserving the location of the input. The following examples demonstrate some characteristics of these networks.

1. Operates at All Input Locations

Set INPUT #6 to "0", and set any other INPUT to "1". Note that maximum CALC value in *Column* K always occurs at location that corresponds to the input.

2. Input Value is Conserved

Successively replace the "1" in INPUT #6 with a "2", "3", "9", "9.4", "0.317". Note the value in K18 is always 5 times the value of the INPUT. Therefore, a series of "divide K*n* by 5" formulas in *Column* M reproduce the input values

3. Multiple Inputs

Begin by entering "1's" at INPUTS #5 and #6. Two maxima now appear as "1.8's" in

OUTPUT’s 5 and 6 in *Column* M.   - a summation effect. Enter another “1” at INPUT #7. A single maximum output is produced at OUTPUT 6. Entering “1's” at more inputs produces a single maximum for an odd number of INPUTS and double maxima for even numbers of inputs. The ability of the correction in *Column* M to reconstruct the value of the inputs has been lost. This is similar to spatial summation effects in sensory systems.

Successively change INPUT #5 to “2, 3, 9". While the location of the maximum output corresponds to the location of the maximum input, it is no longer possible to calculate the value of either input. The larger input has masked the smaller.

4. Spatial Resolution

How far apart must two simultaneous inputs be to avoid interference? Enter “1" at INPUTS #6 and #8. A maximum “8" is produced at the intermediate OUTPUT K20.
To see the limits of spatial resolution, successively enter “1's" at INPUTS #9, #10, #11, #12, while deleting the previous second input. For the maxima to occur in the correct position with a value that can be restored to the value of the corresponding input, requires a gap of 4 inputs. If the second input is substantially larger than the first, complete independence requires a gap of 5 inputs. Adding a layer that produces lateral inhibition between the inputs and convergent calculations can improve spatial resolution, but this usually prevents correcting the output to equal the value of the input - see Appendix **B**. Greater convergence can be obtained by adding more columns of convergent and divergent branching calculations - see Appendix **C**. Successively scanning the INPUTS provides perfect spatial resolution and the correct output value for any number of simultaneous inputs - see Appendix **E**.

APPENDIX B: LATERAL INHIBITION SOMEWHAT IMPROVES SPATIAL MULTIPLEXING

Go to the author's University of Prince Edward Island web page: http://www.islandscholar.ca/fedora/repository/ir%3Anilsson . Open the file "Spatial Multiplexing". Download into a Pro spreadsheet the file *LinC-D2to1LatInhib.qpw*.

On the left, this spreadsheet contains the basic 2-to-1 convergent-divergent network shown in Figure 4 and demonstrated in Appendix A. On the right a second 2-to-1 spreadsheet has added in *Column* V a series of lateral inhibitory calculations on the inputs in *Column* U. These calculations amplify the immediately adjacent input by 2.67, subtract 0.67 times the inputs in the second closest inputs and subtract 0.33 times the input of the third closest inputs. For example: The lateral inhibitory calculation at V24 equals +U24*2.67 plus U22*0.67 plus U26*0.67 plus U20*0.33 plus U28*0.33. In this case a "2" has been entered into INPUT 4 at C16 and U16. *Cell*s U12, U14, U18, and U20 all equal "0", so V16 equals "5.33".

A value of "1" has also been entered at INPUTS 8 & 13 of the BASIC NETWORK on the left. Along with the "2" at INPUT 4, these are copied into the same INPUTS of the LATERAL INHIBITION network on the right. In the basic network on the left, INPUT 8 does not appear in the corresponding OUTPUT 8 or anywhere else in *Column* M. In the network on the right with lateral inhibition, INPUT 8 appears at the corresponding OUTPUT 8, though at a sightly reduced value. Note that OUTPUT 4 is also slightly reduced.

The reader is encouraged to experiment with various input values, locations, and with different lateral inhibitory weights. The accuracy of information transmission varies with no clear optimum for a variety of conditions. It seems likely that more complex animals with spatially multiplexed nervous systems would have developed a further means of handling simultaneous inputs.

APPENDIX C: TRIPLE CONVERGENT-DIVERGENT NETWORK

Start by going to the author's University of Prince Edward Island web page: http://www.islandscholar.ca/fedora/repository/ir%3Anilsson . Open the file "Spatial Multiplexing". Download into a Pro spreadsheet the file *LinC-D8to1.qpw*.

This triple layer model transmits *n* inputs across *n*/8 pathways. On the left or convergent side of the spreadsheet, *Column* B contains inputs at B17, B19, B21, . . ., B120. *Column* D contains the 1st layer of convergent calculations in *Cells* D16, D18, . . ., D256. For example, D20 finds the sum of INPUTS (B17 + 2*B19 + 2*B21 + B23), which is the same as the calculation used *Column* E of the simple model in Appendix **A**. The D20 calculations are repeated in D24, D28, etc. *Column* E contains the 2nd layer of convergent calculations which use the results in *Column* D in the same manner as the *Column* D calculations use the inputs in *Column* B. Thus E22 finds the sum (D16 + 2* D20 + 2*D + D28). *Column* F contains the 3rd layer of convergent calculations that sum the results in *Column* E in the same manner as *Column* E summed the results in *Column* D. For example, F26 equals the sum (E14 + 2*E22 + 2*E30 + E38). The values shown in *Column* F are sent over the pathways illustrated by the lines in *Column* G. These values reappear on the divergent side of the spreadsheet in *Column* H.

Column J contains the 1st layer of divergent calculations. For example, J22 is the sum (H10 + 2*H26) and J30 is the sum (2*H26 + H42). These pair-wise sums are repeated down *Column* J. *Column* L contains the 2nd layer of divergent calculations. L20 contains the sum (J14 + 2*J22). For example, L16 contains the sum (2*J14 + J22). These pair-wise calculations are repeated down *Column* L. *Column* N contains the 3rd layer of divergent calculations. For example, N15 contains the sum (L12 + 2*L16), while N17 contains the sum (2*L16 + L20). These calculations are also repeated pair-wise down *Column* N. The values in *Column* N are the network's outputs.

To see how this network functions, the reader is advised to follow procedures similar to those for the simple convergent-divergent network as discussed in Appendix **A**. One difference between the single and triple models can be seen by entering a "1" successively at inputs #9, #10, #11, #12, #13, and #14. The maximum outputs at corresponding locations N36 to N46 have the following values respectively: 365, 357, 357, 365, 365, and 357. Therefore to obtain the actual value of the INPUTS, the OUTPUTS are divided by either "357" or "365" (the pattern is predictable) to obtain almost accurate output values in *Column* P.

This 8-to-1 convergent-divergent network has poor spatial resolution. There must be a gap of 10 inputs for simultaneous inputs to appear at corresponding outputs and for those outputs to be scalable to the input values. Lateral inhibition can be added by inserting *Column* V from the spreadsheet *LinC-D2to1LatInhib.xls*.

APPENDIX D: RESILIENCE OF SPATIAL MULTIPLEXING

For simplicity, this demonstration uses the non-scanning 16-to-1 spatial array QuattroPro spreadsheet *SurfC-D16-1.qpw* available on the author's University of Prince Edward Island web page http://www.islandscholar.ca/fedora/repository/ir%3Anilsson in the "Spatial Multiplexing" file.

On *Page A*, a value of 7.2 has been entered in INPUT AE30. *Page B* contains the convergent calculations. *Page C* shows the values being sent by the transmission pathways represented in the green cells. The 7.2 INPUT is shown in *Cell* AE30. This value also appears in OUTPUT AE20 on *Page F*.

Go back to *Page C* and delete the green pathway AB27. It is the pathway closest to the location of the 7.2 INPUT AE20. Go to *Page F* and note the network's OUTPUT is now 5.12 in *Cell* AG32. Go back to *Page C* and restore pathway AB27 by copying any other green cell pathway to it.

Now delete the most distant pathway, AJ35, that transmits information from INPUT AE30. Go to *Page F* and note that OUTPUT's AE28 and AC30 (which bracket the position of INPUT AE30) both equal 6.06. Go back to *Page C* and restore pathway AJ35.

To see the effects of losing a converging branch, go to *Page B* and delete the branching calculation in *Cell* AH29. Go to *Page F*, and note OUTPUT AE30 is now 5.79 in the correct location. Go back to *Page B* and restore *Cell* AH29 by copying *Cell* AH21.

Finally, to see the effects of losing a divergent branch, go to *Page D* and delete *Cell* AD33. *Page F* now shows OUTPUT AE28 equals 6.73. Restore *Cell* AD33 on Page D by copying *Cell* AD35.

It is left to the reader to try deleting one or more other branches and pathways. Generally, the closer the damage is to the location of an input, the greater the change in output value and the greater the discrepancy in location. Unless there are several losses, the output continues to bear some resemblance to the input.

APPENDIX E: SURFACE ARRAY SPATIAL MULTIPLEXING

To start, go to the author's University of Prince Edward Island web page: http://www.islandscholar.ca/fedora/repository/ir%3Anilsson . Open the file "Spatial Multiplexing". Download into QuattroPro the spreadsheet *LinC-D2to1.qpw*. This spreadsheet model uses two sets of convergent and divergent branching networks to obtain a 16-to-1 convergence:

1. Go to INPUT *Page A*. To keep the overall size of this spreadsheet modest, inputs are restricted to cells within the green border. Note the value of 7.2 entered at location AE30.

2. Go to the CONVERGENT BRANCHING *Page B*. The inputs from *Page A* are indicated by the small plain cells. The pale blue cells contain the first set of convergent calculations based directly on the inputs. The green cells contain the second set of convergent calculations based on the values in the grey cells. These green cells project directly to the green transmission pathways shown on *Page C*.

An example of the 1st set of convergent calculations is highlighted in the yellow bounded region that extends from K10 to Q16. The calculation performed by the outlined grey *Cell* N13 is highlighted in yellow starting at N1 at the top of the page. The Q, W & E weights of each branch are 0.71, 0.32 & 0.24 as listed in a table at A11 and explained in Figure 7.

An example of the 2nd set of convergent calculations is highlighted in the blue bounded region that extends from F21 to R33. The calculation performed by the outlined green *Cell* L27 is highlighted in blue starting at L2 at the top of the page.

3. *Page C* focusses on the values transmitted by pathways which are represented by the green cells. The smaller numbers are the inputs from *Page A*.

4. On *Page D*, DIVERGENT BRANCHING, cells with the red values draw from the four closest pathway cells in green to do the 1st set of divergent calculations. For example, within the blue bounded area at L11 to T19, the outlined *Cell* at N13 gives the most weight (0.71) to its closest pathway, L11; a lesser weight (0.32) to the two next closest pathways, L19 and T11; and the least weight (0.24) to the furthest pathway, T19. The calculation is shown at the top of the page starting at N2.

The 2nd set of divergent calculations are done by the cells with the values displayed in black using smaller numbers. Within the yellow bounded area at N21 to R25, outlined *Cell* Q24 gives the most weight (0.71) to its closest red-numbered 1st order *Cell*, R25; a lesser weight (0.32) to the two next closest red 2nd order *Cell*s, R21 and N25; and the least weight (0.24) to the furthest 2nd order *Cell*, N21. The calculation is shown at the top of the page starting at Q3.

5. *Page E* shows all the output values obtained from the 2nd order divergent calculation cells in *Page D*. It also has the correction weights needed to convert a maximum output depending on its location so it equals the input on *Page A*.

6. The OUTPUTS *Page F* finds maximum values in the output array on *Page E* and corrects those values based on the weights in *Page E*. For example *Cell* AE30 checks to whether its output equals the largest output within the range of outputs from AA26 to AI 34. If it is the largest out, its value is divided by the corresponding correction weight in *Cell* CC30 on *Page E* and displayed. Non-maximum outputs are not corrected or displayed.

APPENDIX F: SCANNED SPATIAL MULTIPLEXING

Go to the author's University of Prince Edward Island web page: http://www.islandscholar.ca/fedora/repository/ir%3Anilsson . Open the file "Spatial Multiplexing" and download into QuattroPro the spreadsheet *SurfC-D16to1Scan.qpw*.

This spreadsheet is the same as *SurfC-D16to1.qpw* except a new page has been inserted between the INPUT *Page A* and the CONVERGENT BRANCHING *Page B*. The new *Page S* has an empty looking Buffer Array at U20 - AQ42, a Horizontal Scanning Vector at U1 - AQ1, and a Vertical Scanning Vector at A18 - A42. Each cell in the buffer array multiplies: (the value of its corresponding cell in the INPUT array on *Page A*) X (the value in its column of the horizontal scanning vector at the top of the page) X (the value in its row of the horizontal scanning vector on the right). Unless there is a "1" in the proper row and column of the scanning vectors, the value of a cell in the Buffer Array is zero.

Two macro programs successively move two "1's" through each cell in the Horizontal and Vertical Scanning Vectors. Thereby briefly entering the value of each INPUT cell on *Page A* into the Buffer Array. That value then appears at its corresponding location on CONVERGENT *Page B* - briefly but long enough for the rest of the spreadsheet to find the location of the maximum, and enter a corrected value into the OUTPUT *Page E* without interference from other inputs. When other inputs appear briefly, they only produce maxima at their corresponding location in the output array. Thereby, the output array accumulates all the inputs until a new scan is initiated.

To run this spreadsheet:

1. Go to *Page A*. Enter some numbers into the green bordered INPUT array at U20 - AQ42.
2. Go to the scanning page, *Page B*. Press Alt & F2.
3. A "Play Macro" table appears. Left click on "DO" in the Macros/Named Cells list. Left click on "OK" at the bottom.
4. Wait for a "beep".
5. Go to *Page H* to see the scanned outputs.

(Cells on the other pages will be filled with zeros since the last scan resets the scanning vectors.)