

Photometric specification of images

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The traditional perspective on specifying a stimulus' luminosity is to measure its emitted light. Because viewing is the reverse of radiating, an alternative approach measures the light as it is received by the eye or other image sensor – *incident photometry*. From this perspective, I develop a method of specifying stimulus luminosity in terms of the flux incident on noncontinuous detector surfaces whose sensing units map the directional characteristics of an image. Sensing unit size determines how much of the image at its location induces its sensory response. That portion of the image is the sensing unit's *receptive field*. Since sensor response depends on rate of photon impingement, the appropriate photometric measurement is *lumens-per-receptive-field* (lm/rf). 'lm/rf' accounts for pupil area and can be corrected for ocular transmittance. Photometry of images can be completed by specifying the light attenuation needed to reach the sensing system's response threshold.

Keywords: incident light; vision; receptive field; threshold; luminance; retinal illuminance

1. Introduction

This description of how light can be measured in terms of functional image sensing units applies equally to photometric specification of stimuli for vision and radiometric specification of inputs to non-biological image sensing devices. To avoid innumerable parallel references to visual and non-biological systems, the discussion focuses on the visual perspective.

When we think of measuring what we see, we innately assume that our conscious experience is the reality *out there* (see Figure 1). Therefore, when we seek to specify visual stimuli, it seems obvious that we should measure what is *out there*. This positivist view is so fundamental in the training of scientists and engineers, that even those who venture into vision research find it difficult to conceive of any other way to measure the luminous characteristics of the stimuli. Few would disagree with Bodmann (p. 29) that 'Photometry refers to the brightness aspect of visible radiation' [1]. Yet the circularity of light being both a stimulus for vision and also a visual response has led vision researchers to conceptual difficulties [2].

The light-source perspective on photometry is reinforced by traditional illustrations of the Inverse-Square Law of radiation such as Figure 2, which is representative of a number of such figures [3–6]. After seeing the compelling logic only once, it is difficult to think about photometry any other way.

There is an alternative to the classical interpretation of luminous intensity and luminance as a means

of describing light sources. With equal rigor, these measurements describe light as it is incident on the eye [7]. The incident light perspective suggests a functionally more valid alternative to retinal illuminance or Trolands for describing luminosity of the retinal image. To explain this alternative, we begin with the fundamentals of the traditional approach to photometry by considering how the light emitted by a point source is measured.

2. Luminous intensity

The basic setup for measuring luminous intensity involves a light detector located a certain distance in front of a point source as shown in Figure 3. Its numerical response, '*n*', depends on the impingement rate (flux) and wavelength of photons. The detector's spectral sensitivity matches that of the human eye. This enables the detector's response to be calibrated to indicate *lumens* (luminous flux).

2.1. Correcting for distance and size

Tests would quickly reveal that the detector's response varies depending on its distance from the source. Knowing that the source has a steady output, photometrists could have standardized the measuring distance to one meter or yard or cubit – depending on when and where the rules were written. To avoid these

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vagrancies, a simpler solution *corrects* the detector's response for its distance. According to the Inverse-Square Law this correction requires multiplying the detector's response by the distance squared. Doing so results in a constant reading at any distance from the source.

Further consideration suggests that detector response also depends on size of the detector's photo-sensitive area. The bigger this area, the more light it captures. To avoid this problem, photometrists could

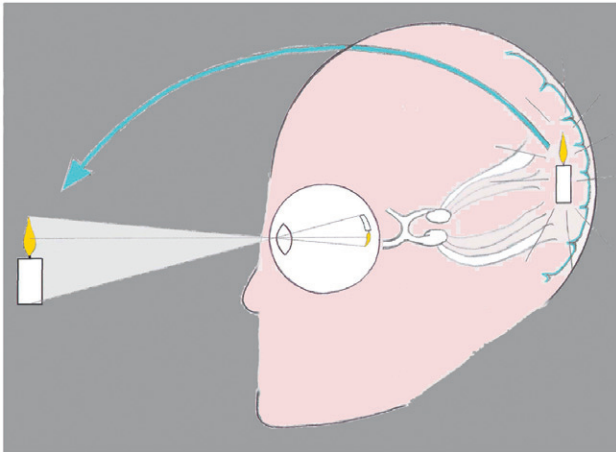


Figure 1. Because what we see appears to be *out there*, we naturally tend to measure what is out there when it comes to specifying the stimuli used in vision research. (The color version of this figure is included in the online version of the journal.)

have chosen a standard detector area. However, this would be problematic given the various sizes of photo-detectors from chemical coatings, to miniature photodiodes, to photovoltaic devices. Again a simpler solution was taken – dividing by a detector's area. These corrections result in the following formula for any detector:

$$\begin{aligned} \text{luminous intensity}_{\text{point source}} \\ = n_{\text{lumens}} \times (\text{distance}^2 / \text{area}_{\text{detector}}). \end{aligned} \quad (1)$$

This formula may not look familiar to regular users of photometry. That is because the *times-detector-distance-squared* and *divide-by-detector-area* corrections can be combined into a completely different single attribute – solid angle.

2.2. Solid angles

Solid angle can be considered a finite expression of direction. While direction is often described as a vector, a solid angle includes a cone-like spread around the particular direction of a vector. Due to the Inverse-Square Law, a directional vector is unsuitable for measuring radiation. As a single ray, it would involve a minute quantity of luminous flux that diminished with distance. A solid angle encompasses a constant amount of luminous flux at any distance.

Solid angle is measured in *steradians*. Geometry dictates that 4π conical steradians complete a sphere of all directions surrounding a point. Accordingly, a

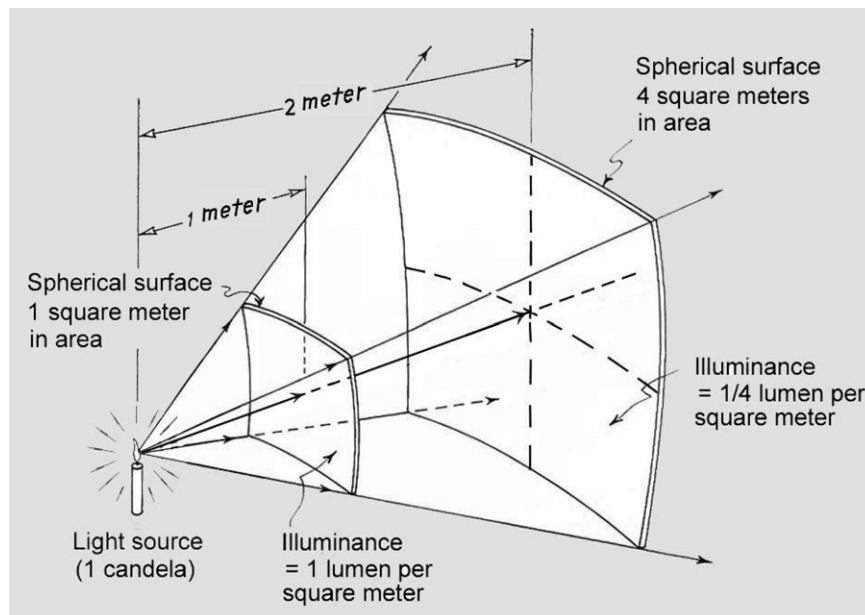


Figure 2. How the Inverse-Square Law of radiation has been widely illustrated. It explains why the strength of light decreases with distance-squared from a light source.

steradian is the conical range of directions that comprise about 1/12th of a sphere. Because of its size, a steradian is not easy to visualize. One can see its approximate size by extending an arm straight out in front. Bend the elbow 90°. Clench a fist. Rotate the forearm about its center. With the other eye closed, the field of view swept out between the knuckles and the inside of the elbow is close (1.2 times larger for me) to a steradian (see Figure 4).

Vision researchers are probably more familiar with the two-dimensional geometry of visual angles. The plane angle within a 1-steradian cone equals 65.54° (see Figure 5).

The solid angle in steradians of a conic section of a sphere is defined as the area of its open end divided by that area's distance-squared from the center of the sphere. Strictly speaking, the open end of a conic section is a spherical surface with all points equally distant from the sphere's center. However, as long as the tangential radius of that opening is less than 1/10th the opening's distance from the sphere's center, the plane area of the open end ($\pi \times r^2$) is accurate to 1% [6]. (To save space, the accompanying figures do not conform to this rule.) Therefore, the solid angle that the detector in Figure 3 subtends with respect to the point source in steradians is:

$$\text{steradian}_{\text{detector}} = \text{area}_{\text{detector}} / \text{distance}^2. \quad (2)$$

2.3. Directional flux

Note that the above formula for solid angle is the inverse of the correction factor that was used to measure luminous intensity. Substituting a 1/steradian term for the ($\text{distance}^2 / \text{area}_{\text{detector}}$) correction factor in

Equation (1) produces the following, familiar, formula for luminous intensity:

luminous intensity

$$\begin{aligned} &= n_{\text{lumens}} \times (\text{distance}^2 / \text{area}_{\text{detector}}) \\ &= n_{\text{lumens}} \times (1/\text{steradian}) = n_{\text{lumens}} / \text{steradian}. \end{aligned} \quad (3)$$

If the detector's distance and area are measured in the same units, and the detector is calibrated in lumens, then the luminous intensity measurement of the point source is in international standard units of *lumens-per-steradian*.

Consider what has been accomplished by this substitution. Dividing a lumens reading by the solid angle over which the measurement was obtained has produced a stable measurement of the point source regardless of distance or size of the detector. In this format luminous intensity clearly embodies that critical characteristic of light which makes it so valuable – its directionality. As a solid angle, the steradian is the finite unit of direction. Thus, lumens/steradian specifies how much light is emitted by a point source in a unit of direction.

The eye is a directional sensor of radiation. What we see depends primarily on flux per direction and not simply on the total amount of flux as indicated by lumens. That is why the lumens/steradian term is the basic unit of photometry and given a name of its own – the *candela*. Initially, objective quantification of light sources required some standard for visual comparison. That standard became the unit of measurement. Organic candles were used as approximate point sources for a standard of luminous intensity. When an electronic standard replaced an actual candle, it was configured to closely resemble the luminous intensity of the previous candle standard [8]. Today's standard unit of luminous intensity is called the candela to

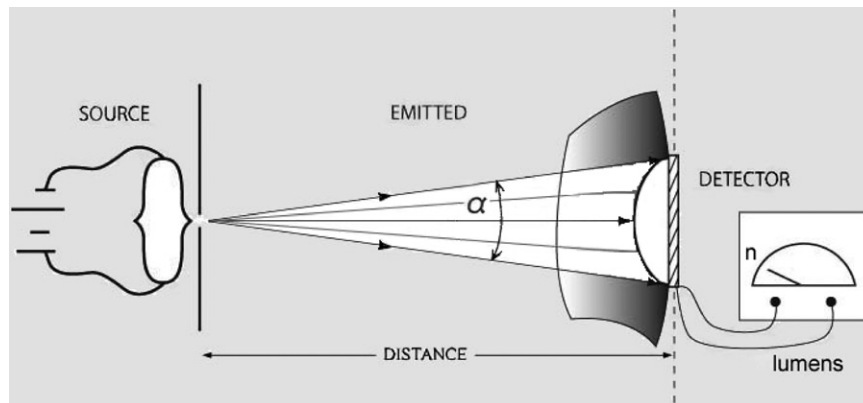


Figure 3. The meaning of *luminous intensity* is represented by measuring the luminous flux emitted from a point source towards a detector. The detector's response of n lumens depends on its distance and its area, which together define the solid angle α that the detector subtends with respect to the source. (Based on [7], Figure 2.)

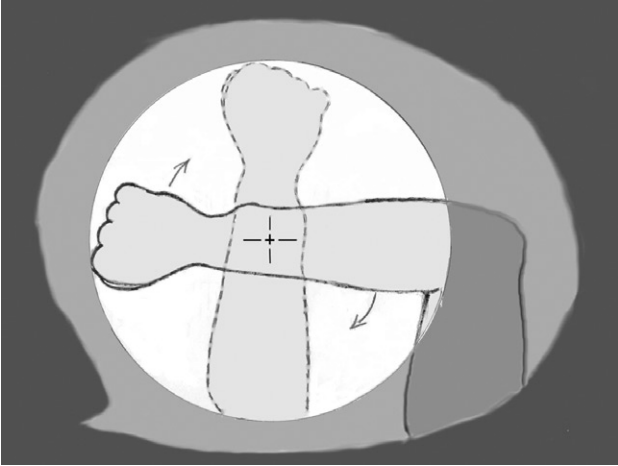


Figure 4. How to directly see a field of view of that corresponds to approximately 1 steradian – light circular area. See the text for instructions.

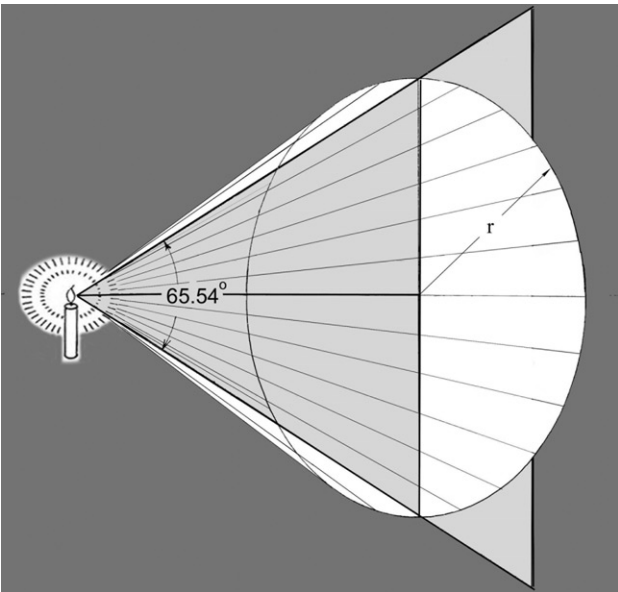


Figure 5. A cone of light equal to 1 steradian is selected from a candle's radiation. A vertical plane bisecting this cone has a plane angle of 65.54° . (The circular opening of the cone has been horizontally compressed into an ellipse to create a three-dimensional effect which still enables an accurate representation of this angle in the vertical plane. A variation of the Ponzo illusion plus expected perspective makes the left half of the 'circle' appear larger than the right half.)

distinguish it from the earlier, slightly different unit based on and named for a candle.

On that basis, the lumen is defined as a 'candela-steradian' – namely in terms of the luminous intensity of a point source times the solid angle that the measuring detector subtends with respect to the source. This definition of a simpler unit in terms of

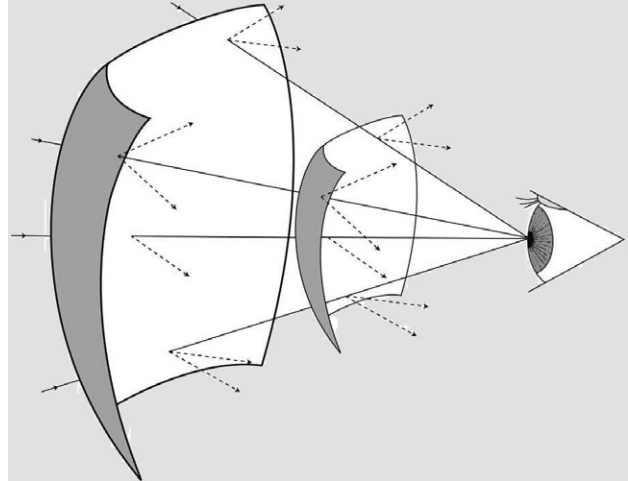


Figure 6. Viewing can be considered as the reverse of radiating. A spherical surface source emitting one lumen per meter-squared close to the eye produces the same effect as a larger surface emitting $1/4$ lumen per meter-squared at twice the distance.

a more complex unit reflects the visual basis of photometry.

3. 'Reverse' photometry

It is the directional attribute of viewed light which potentially makes it extremely valuable as a source of information for animal life. Reflected more or less by everything in the environment, light incident on the eye from various directions reveals the presence, location, and nature of most things around us.

Figure 6 illustrates the general situation of light reaching an eye from the surrounding environment. It shows that *viewing* is the reverse of *radiating*. Since the laws of physics are reversible with respect to the direction of light, measuring incident light with respect to the eye must obey the same rules as measuring light emitted by a source. This also enables using the same photometric units for both types of measurements. Thank goodness!

3.1. Incident luminous intensity

Starting with the simplest photometric measurement that involves the directional characteristic of light, consider the eye as a point detector, which it effectively is for many purposes at distances over 3 m. Figure 7 depicts the situation for measuring the luminous intensity incident on such a detector. A certain emitting or diffuse reflecting, uniform source radiates light in all directions. Some of that light is incident on the point detector at a distance from the source.

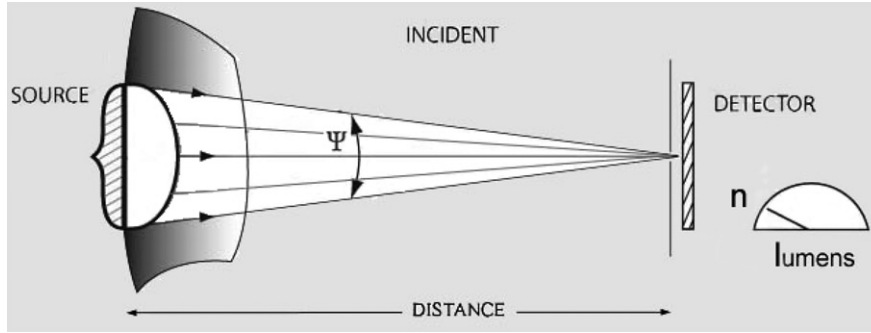


Figure 7. An alternative meaning of *luminous intensity* is represented by measuring the luminous flux from an extended source over a solid angle ψ incident on a point detector. At a certain distance the detector produces a response of n lumens. The arrows indicating the solid angle Ψ bow like wave fronts to indicate the direction that the light travels. (Based on [7], Figure 2.)

Looking at the source through the pinhole in front of the detector in Figure 7 would reveal that the source appears equally bright regardless of its distance and size. However, the detector's response varies inversely with source distance-squared and directly with the source's area. To obtain a photometric measurement that is consistent with what we see, the detector's response must be multiplied by the distance-squared and divided by the source's area. Therefore, the formula for a reading of incident luminous intensity is:

$$\begin{aligned} \text{incident luminous intensity} \\ = n_{\text{lumens}} \times (\text{distance}^2 / \text{area}_{\text{source}}). \end{aligned} \quad (4)$$

Similar to the situation for measuring point sources, the $(\text{distance}^2 / \text{area source})$ correction equals the inverse of the solid angle that the source subtends with respect to the detector. Thus:

$$\begin{aligned} \text{incident luminous intensity} \\ = n_{\text{lumens}} \times (1 / \Psi_{\text{solid angle of source}}). \end{aligned} \quad (5)$$

Provided that the detector response is calibrated in lumens, the result then becomes lumens/steradian – the familiar candela. In this application, however, the candela is a measure of ‘incident’ luminous intensity. The substitution of solid angle for distance and area measurements shows that this luminous intensity refers to the luminous flux received per unit direction. The incident luminous intensity from an approaching source would remain constant despite an increasing detector response, because that response gets divided by a correspondingly larger solid angle in calculating luminous intensity.

3.2. Incident luminance

Incident luminous intensity can be used to describe visual stimuli for certain animals like the nautilus and certain optical devices like the pinhole camera which

use pinhole lenses to obtain directional sensitivity. However, most eyes and image sensing devices have an extended entrance pupil to gather more light. Figure 8 depicts the situation when an extended detector receives light from an extended source located at a certain distance. The numerical response, n , will vary depending on the solid angle that the source subtends with respect to any point on the detector surface. As the sum of the responses at each of these points, the detector's response will also increase with the detector's area.

To our eyes, large light sources or surfaces may appear to be slightly brighter or dimmer than small surfaces of equal luminance. These perceptual changes are small compared to the effect of the Inverse-Square Law. They are attributable to visual, not photometric processes. The primary issue for incident photometry is to measure the light reaching the eye in a manner that corresponds to the predominantly consistent brightness we experience regardless of a source's visual angle. If this were not possible, photometry would be of little interest to vision researchers.

Doubling the solid angle of a viewed source can be expected to double the response, n , from the detector in Figure 8. As was the case with incident luminous intensity, the effect of this solid angle is corrected by dividing by the solid angle, Ψ , that the source subtends with respect to the detector. There remains the matter of the effect of detector size. Rather than standardize detector size, dividing the detector's response by the detector's area enables the measurement formula to apply to light detectors of various areas:

$$\begin{aligned} \text{incident luminance} \\ = (n_{\text{lumens}} / \Psi_{\text{solid angle of source}}) / \text{area}_{\text{detector}}. \end{aligned} \quad (6)$$

As was the case with incident intensity, incident luminance remains constant regardless of the source's distance. Specifying the solid angle of the source in steradians, substituting candela for lumens/steradian,

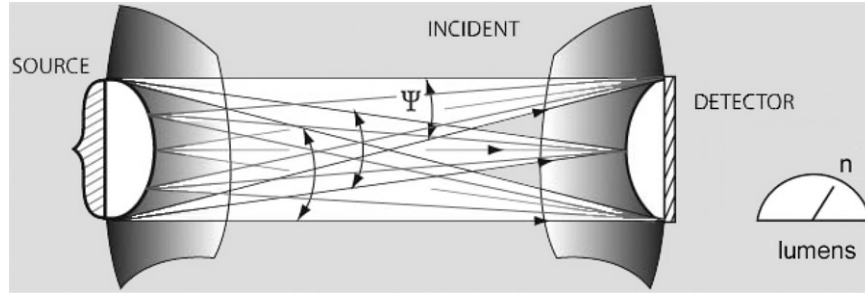


Figure 8. Measuring the luminance incident on an extended detector from an extended source. Each point on the detector receives light from the source over a solid angle ψ to produce a total reading of n lumens. (Based on [7], Figure 3.)

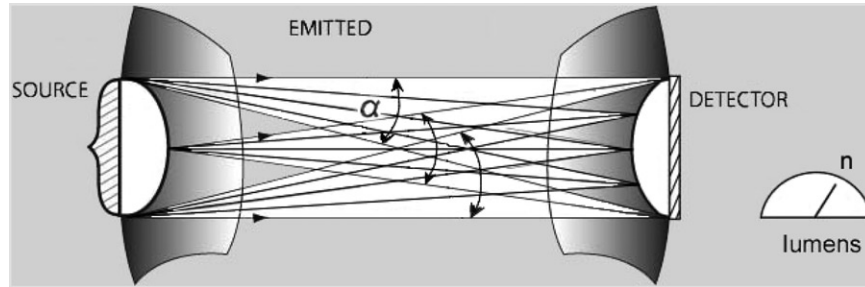


Figure 9. Using a detector having an extended photosensitive surface to measure the luminance emitted by an extended source. Each point on the source emits light over a solid angle α onto the detector to produce a total reading of n lumens. Due to the Reversibility Principle, incident and emitted luminance measurements yield the same result. (Based on [7], Figure 3.)

and specifying the detector area in meters-squared results in the familiar format:

$$\text{incident luminance} = \text{candela}/\text{meter}^2 \quad (7)$$

with the *meter*² term referring to the detector and the *candela* term containing the solid angle that the source subtends with respect to the detector.

Rather than specifying some distant light source, incident luminance would seem a more direct way to specify visual stimuli. Since the units of incident luminance are the same as those for emitted luminance, vision researchers may wonder if it is possible to convert their photometers to read incident luminance.

3.3. Nothing new in front of the candle

Figure 9 shows the situation for measuring the classical emitted luminance of an extended light source. To describe the source independently of the distance and size of the detector, the reading n_{lumens} is multiplied by the distance and divided by the detector area. Alternatively, divide n by the solid angle, α , that the detector subtends with

respect to the source. To make this result independent of the size of the source, it is further divided by the source's area:

$$\text{luminance}_{\text{emitted}} = (n_{\text{lumens}}/\alpha_{\text{solid angle of detector}})/\text{area}_{\text{source}}. \quad (8)$$

Since a lumen/steradian is a candela, the end result is, of course, in units of candela/meter².

While the formulas for emitted and incident luminance are similar, they differ in referring to different solid angles and areas as emphasized in the following diagram:

MEASUREMENT	SOLID ANGLE	AREA
EMITTED:	detector	source
INCIDENT:	source	detector

Notwithstanding these differences, matters are not as different as they appear. Breaking down that 'per solid angle of detector' component in the emitted luminance formula produces:

$$\begin{aligned} & (n_{\text{lumens}}/\text{solid angle}_{\text{detector}})/\text{area}_{\text{source}} \\ & = (n_{\text{lumens}}/(\text{area}_{\text{detector}}/\text{distance}^2))/\text{area}_{\text{source}}. \end{aligned} \quad (9)$$

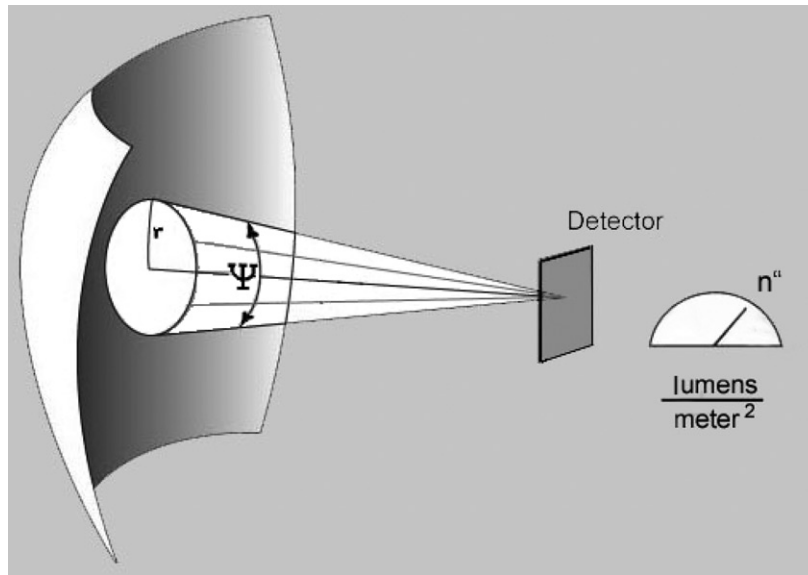


Figure 10. How to measure luminance with an illumination meter. A luminous aperture of radius r is located at a certain distance from the detector surface. As long as the aperture's diameter is less than 1/10th the distance from the detector – not as illustrated, the solid angle Ψ equals the aperture's plane area ($\pi \times r^2$) divided by the distance squared. If the meter is calibrated in lumens/meter² (lux) and reads n'' , the incident luminance is then (n''/Ψ) candelas per meter-squared. (Based on [7], Figure 4.)

Rearranging the divisions reveals that:

$$\begin{aligned} &\text{emitted luminance} \\ &= n_{\text{lumens}} \times \text{distance}^2 / (\text{area}_{\text{detector}} \times \text{area}_{\text{source}}). \end{aligned} \quad (10)$$

Similarly:

$$\begin{aligned} &\text{incident luminance} \\ &= n_{\text{lumens}} \times \text{distance}^2 / (\text{area}_{\text{source}} \times \text{area}_{\text{detector}}). \end{aligned} \quad (11)$$

Because the same breakdown and rearrangement can be done with the formula for incident luminance, the two photometric measurements are equal! Unbeknownst to many, their photometric readings can be interpreted as either emitted or incident luminance.

3.4. Cheap way to measure luminance

Many vision researchers do not have the sort of funding that enables purchasing \$10,000-plus photometers to measure the luminance of a particular visual stimulus. The incident perspective on photometry shows how luminance can be measured with a simple photographic light meter. These are readily recalibrated to measure the illuminance in terms of lumens/meter² – also called 'lux' [9].

Incident luminance from a source is measured in candela/detector area, which is also:

$$\begin{aligned} &\text{incident luminance} \\ &= (n_{\text{lumens}} / \Psi_{\text{steradians of source}}) / \text{square-meters}_{\text{detector area}}, \end{aligned} \quad (12)$$

where the Ψ steradians refer to the solid angle that the source subtends with respect to the detector (see Figure 10). Since it does not matter in which order the divisions are done, the above equation is equivalent to:

$$\begin{aligned} &\text{incident luminance} \\ &= (n_{\text{lumens}} / \text{square-meters}_{\text{detector area}}) / \Psi_{\text{steradians of source}}. \end{aligned} \quad (13)$$

That *lumens/square-meters* detector area term is 'illuminance'.

Ostensibly, all that is involved to find stimulus luminance is measuring the illuminance produced by the stimulus at the location of the eye and dividing by the solid visual angle of the stimulus. However, two other matters need to be considered. (1) Lacking a lens, an illuminance meter is not directionally selective of the light to which it responds. Therefore, one must ensure that only light from the stimulus reaches the detector. (2) Calculating the solid angle of a stimulus is easy enough if it is round. If it is not round, use a slightly smaller circular aperture and place it directly in front of the stimulus. Then calculate this aperture's solid angle. A little work and some math can save thousands of dollars.

Based on this relation between luminance and illuminance, one can inexpensively build a combined luminance and illuminance reference source to test the stability of both photometers and light meters [10]. A major concern with a ‘home-made’ standard is whether a change is due to a change in sensitivity or a change in the source. A combined standard enables comparing a photometer’s sensitivity with the sensitivity of a less expensive light meter. Since it is unlikely that two of the three factors would change at the same time, one can be reasonably certain which one has actually changed.

4. Specifying images

What could be better than specifying the light incident on the eye or other image sensing device? For vision research, specifying the amount of light in the image on the retina itself would provide the closest basis for linking the physical to the physiological events that lead to perception. However, retinal image characteristics depend on more than what meets the eye.

Illuminance is the amount of luminous flux in transit per unit area – flux density. It is the only photometric measurement commonly used to refer to incident light. (When referring to light emitted per area of a source, the term *luminous exitance* is often used instead.) At first glance, illuminance may seem the most appropriate photometric measurement for vision research. Whatever flux density reaches the eye must be proportional to flux density of the retinal image – apart from the pupil and intraocular transmission effects. The problem with illuminance is that it is an average measure of flux density. It does not distinguish a small strong light source from a large weak one. Such discrimination requires a directionally sensitive measurement, which is why luminance is preferred over illuminance for vision work [11].

However, specifying luminous flux per solid angle of the source per detector area ignores the fact that most image sensors are not continuous sensing surfaces like a photodiode. Rather, the eye and most other image sensors consist of an array of individual receptors. Therefore, the luminous effectiveness of images would be more accurately specified in terms of how much of the image falls on a representative sensing unit within that array. To see how this geometry can be applied photometrically, we begin with the image produced by a pinhole lens.

4.1. Luminous flux in a pinhole image

Figure 11 shows a pinhole lens forming an image of an extended uniform light source. Not shown is a detector

which when placed directly behind the pinhole, as in Figure 7, indicates a luminous flux of n lumens. The incident luminous intensity is therefore $n_{\text{lumens}}/\Psi_{\text{source solid angle}}$. In forming an image behind the pinhole, this luminous flux gets distributed over the solid angle Ω that the image subtends with respect to the pinhole.

The effectiveness of an image depends on the amount of flux it provides to individual receptors in their respective portions of the image. This is true whether the image is focused on the chemical grains of a film, on electronic pixels of a digital image sensor, or on cone and rod cells of the retina. Each receptor (only one is shown) in Figure 11 receives a portion of the flux spread over the solid angle Ω . A receptor that subtends a solid angle ω with respect to the pinhole receives only (ω/Ω) th of that flux.

For the human eye, the solid angle that a single cone subtends with the center of the pupil can be calculated from its area and distance from the effective center of the lens–pupil–cornea combination. Taking $2.5\ \mu\text{m}$ as a representative cone diameter [12] and using the 22.6 mm distance of a retinal receptor from the second principal point of the eye gives a value of 1×10^{-8} steradian for ω . (This corresponds to a plane angle of 23 seconds.) Back-tracking the rays in ω through the pinhole and out to the source produces a solid angle ψ which is the visual field of a cone. Because congruent solid angles are equal just as are congruent plane angles, $\Omega = \Psi$ and $\psi = \omega$. If the pinhole in Figure 11 represented a pupil of the human eye, the luminous flux incident on any cone within that image would equal:

$$(1.0 \times 10^{-8} / \Psi_{\text{steradians of source}}) \times n_{\text{lumens}} .$$

As explained in Section 2 above, $n_{\text{lumens}}/\Psi_{\text{steradians}}$ is constant regardless of the source’s distance. It follows that flux incident on a receptor is also constant regardless of source distance, but there are two prerequisites: (1) as a source becomes more distant, its image gets smaller. Therefore, the receptor must be one that remains fully within the image. (2) The angle Ψ that the source subtends with respect to the eye must be greater than the visual field ψ of the receptor.

4.2. Flux in a focused image

Pinhole lenses and ray diagrams are good for working out the basics of optical designs. However, apart from rare systems that actually use pinhole lenses or pinhead mirrors, most images are formed by lenses or mirrors that gather many rays from each point on a distant source. Figure 12 illustrates a system similar to that in Figure 11, except an extended lens focuses the image as

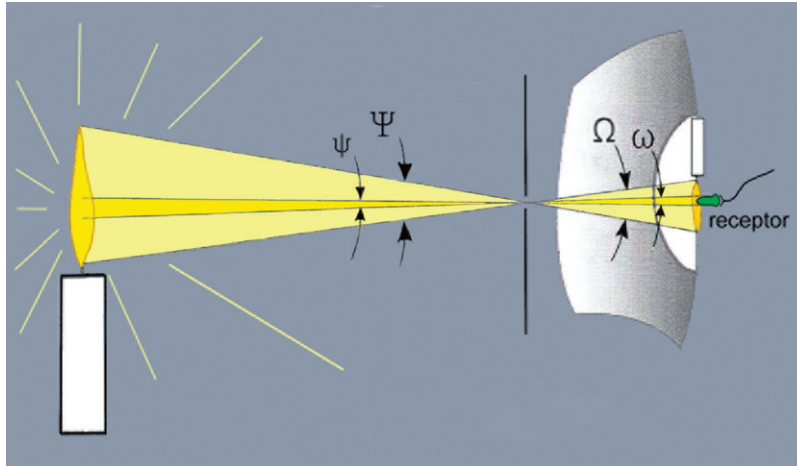


Figure 11. How to describe the luminous flux in terms of image receptors. A pinhole lens is shown forming an image of a candle flame on a surface that contains a multitude of sensors. Ψ is the solid angle that the flame subtends with respect to the pinhole. Ω is the solid angle of the flame's image with respect to the pinhole. The solid angle that a single sensor subtends with respect to the pinhole is ω . That sensor has a field of view through the pinhole which is shown as the solid angle ψ . (The color version of this figure is included in the online version of the journal.)

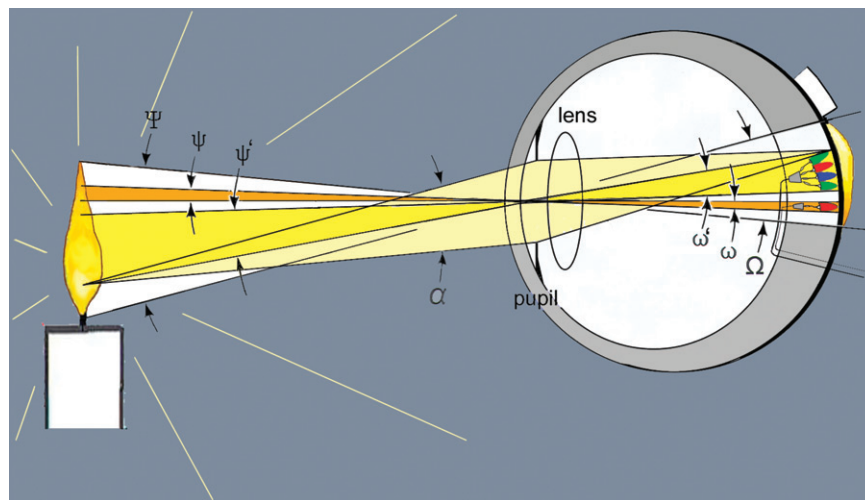


Figure 12. Illustration of a schematic eye forming an image of a candle flame on the retina. All rays admitted from any point on the flame such as those within the solid angle α are focused by the extended lens onto a corresponding point on the retina. Ψ is the solid angle that the flame subtends with respect to the first principal point of the eye's compound lens system. Ω is the solid angle of the flame's image with respect to its second principal point. The solid angle that a single receptor subtends with respect to the second principal point is ω . That receptor has a field of view which is shown as the solid angle ψ . Also shown are several receptors connected to a single retinal ganglion cell. This cluster of receptors subtends a solid angle ω' with respect to the second principal point. That ganglion cell has a receptive field shown by the solid angle ψ' . (The color version of this figure is included in the online version of the journal.)

in the eye. The effective entrance aperture area of this lens is defined by a pupil. Again it is assumed that a detector with the same area as the pupil was placed at the pupil's location and produced a reading of n lumens – the total luminous flux entering the eye.

Due to refraction at its curved surfaces, the lens directionally transmits incident rays so that all rays admitted from any point on the source (for example,

those rays within the solid angle α) fall on a corresponding point of the retina. Despite the lens' extended area, it distributes the luminous flux as if all the flux originated from its center – as a first approximation. (More about this later.) On that basis the luminous flux of n_{lumens} can be considered to be spread over the solid angle Ω as shown in Figure 12. Similar to the case with a pinhole lens, this makes it

easy to specify what portion of that flux reaches a particular receptor in terms of that receptor's solid angle ω with respect to the eye's second principal point. Since the internal solid angle Ω equals the measurable angle Ψ that the source subtends with respect to the lens, once again:

$$\text{flux impinging on a receptor} = n_{\text{lumens}} \times (\omega/\Psi), \quad (14)$$

where n was measured at the location of the entrance pupil by a detector whose area equals that of the aperture. The solid angle ω of the receptor is determinable by anatomical measurements. Because ω equals ψ , Equation (14) also means that the total luminous flux, n is reduced by the ratio, ψ/Ψ , of the receptor's receptive field to the source's solid angle. (When the sizes of round stimuli and round receptive fields are known in terms of plane degrees, the square of their ratio will equal the ratio of their solid angles.)

4.3. Receptive field

As a source moves away from a detector, the light from that source decreases with distance squared, but simultaneously, the image of the source decreases in area with distance squared. Therefore, the incident flux per unit area of the image remains the same. At some distance the area of the image will equal the area of a receptor. At still further distance the image of the source becomes smaller than the receptor, and the total flux impinging on the receptor decreases. Thus, beyond a certain distance that depends on the source's size, receptor response will decrease and the source will appear dimmer.

In the central fovea, a single photoreceptor may contribute somewhat independently to the final visual image (see Figure 12). However, retinal ganglion cells typically collect input from two to a hundred cone receptors over an area whose visual field could extend to some 2.66×10^{-5} steradian – a 20 minute plane angle. The axons of these ganglion cells represent individual pathways to the visual cortex. Therefore, the receptive area of a ganglion cell may be the best estimate of the minimal image area at which source brightness remains constant. This conclusion is consistent with perceptual measurements of Ricco's law – the area over which further increases in stimulus area do not increase detection against a background [13].

Loosely speaking, the 23 second to 20 minute visual angle pathways are akin to pixels in the visual image. Consequently, when a circular target moves away so far that it subtends a plane angle smaller than 20 minutes to 23 seconds, it will lose brightness.

Since Ricco's area increases by a factor of 10 and more with increasing retinal eccentricity, the distance where that begins to happen will depend on the retinal location of the target's image and on the observer's state of light adaptation.

This measurement will be referred to as *lumens-per-receptive-field* (lm/rf). *Receptive-field* rather than *receptor* is suggested because in many circumstances the critical area for spatial integration of luminous flux may encompass several to hundreds of receptors [14] as indicated in Figure 12 by the cluster of receptors whose responses converge on a single ganglion cell. For human vision, this area would be determined psychophysically. It will depend on the stimulus luminance and size, field luminance, and the observer's adaptation state [15]. Being more general, 'receptive field' also works with non-biological imaging devices to specify *Watts-per-receptive-field* where the 'receptors' may be clustered and referred to by various terms such as *pixel*, *sensor*, and *transducer*.

4.4. Entrance pupil

As described so far, the measurement of lumens-per-receptive-field has involved a hypothetical detector whose area equals that of the entrance pupil of the eye or other image forming system. In the human eye, pupil area is reflexively varied by a factor of 16 to control the amount of light in the retinal image. While a detector with a variable aperture is certainly feasible, it could be inconvenient. Instead, we consider how to take entrance pupil into account using conventional illumination and luminance meters.

A simple illuminometer that reads in lumens per meter² (lux) is sufficient. First ensure that only light from the stimulus reaches the detector. Multiply by its reading, n in lumens/meter², by the pupil area in meter² to obtain the amount of luminous flux passed by the pupil. Then divide the result by the ratio of receptive-field solid angle ω to stimulus solid angle Ψ – both measured in steradians as explained in Figure 12 above. When the receptive field is that of a typical receptor in the fovea, the formula for lm/rf corrected for pupil area is:

$$\text{lm/rf} = n'_{\text{lumens per meter}^2} \times \text{pupil area}_{\text{m}^2} \times (1.0 \times 10^{-8} / \Psi_{\text{stimulus}}), \quad (15)$$

where 1.0×10^{-8} is ω , the receptive-field solid angle for a single foveal receptor.

To avoid eliminating all other sources of light as required when using the illuminance measurement method described in the previous equation, lumens-per-receptive-field can also be calculated using a photometer. Multiply the photometer's incident

luminance reading L in [(lumens/source-solid-angle)/detector area] by the pupil area, and by the visual field of view in steradians of the receptive field ω .

$$\begin{aligned} & \text{lumens-per-receptive-field} \\ & = L_{\text{luminance}} \times \text{pupil area}_{\text{m}^2} \times \omega_{\text{receptive field steradians}}. \end{aligned} \quad (16)$$

To use Equation (16), vision researchers would set ω to values ranging from 1×10^{-8} for images in the human fovea to 2.66×10^{-5} for images in the periphery where the receptive field subtends a 20 minute plane visual angle.

Most published vision research has specified the luminous effectiveness of stimuli in terms of candela-per-meter-squared. This will continue since it is the International Standards Association's unit of luminance and since luminance can be directly measured with a photometer. To facilitate comparison with most previous and future research, the lumens-per-receptive-field specification should be accompanied by the measured (or derived) luminance, and also by the assumed receptive field size and the pupil diameter. For comparison with luminance specifications that lack the accompanying pupil diameter, it may be possible to estimate the equivalent lm/rf using Equation (16): provided that the stimulus is large and steady or the background luminance is specified, the relation between luminance and pupil diameter (see [16] Table 14, p. 106) could be used.

4.5. Other measurement considerations

It has long been recognized that the effects of pupil area on brightness hinders comparing results from studies using different stimulus luminances or different effective pupil diameters. In 1917 to help solve this problem, Troland suggested multiplying the stimulus luminance, L , by pupil area in square millimeters [17]. The result is called 'retinal illuminance' and specified in units now named *Trolands*:

$$\begin{aligned} \text{retinal illuminance} & = L_{\text{candela per meter}^2} \times \text{pupil area}_{\text{mm}^2} \\ & = \text{Trolands}. \end{aligned} \quad (17)$$

This is not the same concept of 'illuminance' as the illuminance specified in units of lumens/meter². Dimensionally the *meter*² and *millimeter*² terms cancel leaving a result in *candela* – *micro-candela* to be exact, which is a measure of incident luminous intensity. However, the result is proportional to retinal illuminance, which was Troland's intent. Pupil area divided by the pupil-to-retina distance-squared is the same as the solid angle that the pupil subtends with respect to the retina. Since the Troland already contains the

pupil area, all that is needed to obtain actual retinal illuminance in lumens/meter² is to divide Trolands by this distance squared [18]. Based on a 16.67 mm effective focal length of the human eye, Burns and Webb multiply Trolands by 0.0036 [my correction] to obtain retinal illuminance in standard units of lumens/meter².

Though corrected for pupil area, neither retinal illuminance nor lumens-per-receptive-field provides an absolute measure of retinal stimulation. Of the measurable flux incident on the eye, the number of photons reaching the receptors is reduced by several factors in addition to the pupil [19]. A cosine correction is needed to account for the greater reflectance of off-axis rays by the cornea or other focusing lens. There is general absorption and scatter of photons by the various ocular media – especially the lens and retinal layers in front of the receptors. Finally, waveguide properties of the receptors themselves result in a Stiles–Crawford effect that reduces the effectiveness of larger pupil diameters. These effects could be accounted for by a correction factor that represents their sum. Charman suggests that for large uniform stimuli, variations of some of these factors with pupil area tend to cancel each other [20]. For further discussion of the retinal illuminance question, see Boynton [21], LeGrand [16], and Levi [22].

5. Photometry relative to threshold

The most direct way to measure the luminous effectiveness of an image is to use micro-photodetectors in the retina itself – thereby taking into account all intraocular effects. Connecting these to a computer would enable taking into account the receptive field effects. This method of measuring incident luminous intensity was first used by Hipparchus in 127 BC to specify the brightness of stars. Still used today, his stellar magnitude scale demonstrates that psychophysical scaling can produce quantitative results whose reliability has withstood the test of time longer than any other measurement system. The ready availability of various magnitude reference sources that have been stable for over 2000 years doubtlessly contributes to its success.

In 1697 a Parisian friar, Francois-Marie, came up with an objective way to quantify image brightness without the use of reference light sources [23]. Francois-Marie added glass plates to a viewing tube until a light was no longer visible. This method defines luminous effectiveness in terms of the attenuation in luminous flux (usually expressed in terms of log density) required to reduce a stimulus to threshold – the borderline between visibility and invisibility.

In effect, Francois-Marie reversed Hipparchus' method by relying on a standard radiometer – the human eye. Indeed, the reliability of Hipparchus' stellar magnitude scale establishes the validity of Francois-Marie's genetically standardized photometer. In doing so, Francois-Marie scooped by 300 years the 'latest' developments in photometry which define the candela in terms of standard radiometers [24,25]. With the ready availability of stable neutral density filters and wedges that attenuate light from 0.1 to 6.0 log, this method can readily be employed today in a variety of stimulus presentation situations. (Carbon-based filters are preferable to partial-reflectance filters, especially when several filters are used together.)

The choice of threshold will depend on the visual task. Photopic thresholds can be defined either in terms of foveal viewing or perception of color with chromatic stimuli. Using the method of limits or method of adjustment with trained dark-adapted observers screened for normal color vision and acuity, I have found individual differences in photopic thresholds of around 0.2 log and a reliability better than 0.1 log over a 3 year period.

It is a wonder that this method of photometric specification of stimuli is seldom reported. This is the only method that takes into account all intraocular effects and other individual differences such as the effects of age [26]. Given the increasing research emphasis on linking the physiological and cognitive aspects of vision, this subjective measurement of image effectiveness seems a suitable (perhaps even necessary) compliment to the instrument-based measurements of luminance and lumens-per-receptive-field.

The high quality control of today's electronic image sensors would seem to warrant a system's threshold approach as well. The basic procedure would involve adding neutral density filters at the device's entrance pupil until response to the distal input falls below the system's noise level. Since inherent noise is measurable in rms volts, a consensus for threshold such as a signal 50% greater than the noise level would seem feasible.

6. Conclusion

Imagine the plight of a Renaissance philosopher who had to hold parchments close to a candle to obtain enough light to read, but could see that the candle was just as bright further away. Kepler formulated the Inverse-Square Law of radiation in 1604, but empirical proof was more difficult. It was not until 1725 that Bouguer quantitatively verified that illumination decreased with distance squared [23]. To obtain sufficiently accurate measurements, Bouguer used the eye to compare the illumination from a moveable candle to the illumination from the sun and moon.

This method reduced the role of the eye to a mere null instrument for measuring light. When Thompson applied Bouguer's method to optimize the design of lamps in 1789, photometry became dedicated to specifying light sources [27].

The possibility of replacing the eye altogether arose when Elster and Geitel invented a photoelectric cell in 1889 [28]. By the mid-1900s highly stable electronic light detectors became available [29], and vision researchers soon replaced eye-based instruments like the MacBeth photometer with photoelectric ones. Still more recently, Canada, the United States and other countries have specified the candela in terms of a standard radiometer [24,25]. Thereby photometry has returned to its roots as a measure of incident light. However, the role of the eye seems all but forgotten except in terms of its spectral sensitivity.

In assessing the visual world *out there*, truly 'We are in contact only with photons' [30]. Yet there is more to it than that. The eye is not just a photo-sensitive organ, it is an optical device with a lens that distinguishes those photons according to their direction of incidence and an aperture that limits how many of those photons are admitted. Accordingly, photometric specification of visual stimuli requires taking both directionality and aperture into account. As a measure of the luminous flux density per solid angle of incidence, incident luminance only partially takes the directionality into account. It does not take into account how the directionality of incident photons is distinguished nor how many photons are admitted.

Depending on their direction, incident photons are focused onto different receptors to form a mosaic image. The luminous effectiveness of that image depends on the response of those individual receptors – not on the average directional flux density specified by luminance. How much luminous flux impinges on any receptor depends on receptor size, which determines the range of directions over which it receives light. This range of directions is the solid angle of the receptor's field of view through the lens (represented in Figure 12 by the solid angle ψ) which is the same as the solid angle ω that the receptor subtends with respect to the center of the lens.

Since photons usually radiate from their source, the number captured by the eye from each direction depends on the solid angle that the pupil subtends with respect to the source – represented by Ω in Figure 12. Lumens-per-receptive-field takes the light gathering capacity of the pupil into account by multiplying the incident luminous flux density by pupil area. Corrections for the transmission of the ocular media and effectiveness of different angles of incidence on the lens and retina are also possible, but go beyond the present scope.

Further specification of image effectiveness requires a different, though hardly new, photometric approach. It involves measuring how much the luminosity can be reduced to reach a system's response threshold. This defines the luminosity of an image in terms of the sensitivity of the image detecting system itself. Image luminosity in log density above threshold takes into account all ocular, receptor, neural and/or other system characteristics, including individual differences. Together with lumens-per-receptive-field, luminosity above threshold fully restores the eye and brings other image sensing devices into photometry.

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